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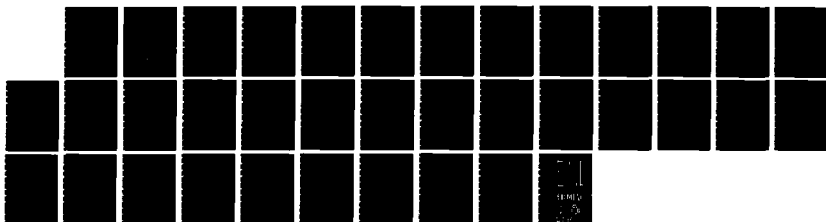
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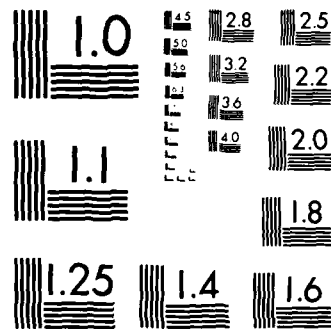
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# High Power Microwave Plasma Pulse Compression

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*High Power Electromagnetics Branch  
Plasma Physics Division*

AD-A163 298

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# HIGH POWER MICROWAVE PLASMA PULSE COMPRESSION

## I INTRODUCTION

Recently there has been a great deal of effort going into producing high power microwave pulses. If a device produces only raw power, such as a vircator or beam plasma source, handling or conditioning the microwave pulse is difficult. However, if there is good control of the pulse parameters one can consider pulse compression, the subject of this paper. With a high power wide band amplifier, one could produce frequency chirped pulses and then compress them by propagating them through some sort of dispersive element. If the dispersive line power capacity exceeds the saturated level of the amplifier, power multiplication to very high levels are possible. Recently, free electron lasers have demonstrated wide-band, high gain amplification at very high power.<sup>1,2</sup> High power pulse compression appears possible if a suitable dispersive device can be found.

The original methods to compress radar pulses made use of the dispersive properties of waveguide modes.<sup>3</sup> Modern radars generally handle the pulse compression in the receiver at low power using lumped circuitry.<sup>4</sup> However, for some applications it is desirable for the outgoing (high-power) pulse to be compressed in time and power multiplied. Pulse compression could be handled at high power if a high-power efficient (non-lossy) pulse compressor could be found. We propose that a plasma filled waveguide would be able to handle high-power pulse compression. Since it is already ionized, it can handle very high power and it is very dispersive near one of its natural

frequencies. For example, a short ( $\approx$  meter) length system can provide low-loss, roughly hundred-fold pulse compression of a 20 nsec, 10% bandwidth x-band microwave pulse.

Section II describes the plasma pulse compression of a chirped pulse output of a high power amplifier. Section III shows that the pulse of a coherent oscillator can also be compressed by a plasma filled waveguide. The idea is to change the plasma properties in time, as the pulse enters, so that a frequency chirp is artificially induced after the fixed frequency pulse has been generated. This is to take advantage of the fact that at high power, at least at X-band and above, oscillators provide both higher power and higher efficiency than amplifiers. Recently, high power oscillators have demonstrated both long time, single mode operation at 20 MW,<sup>5</sup> and also have demonstrated phase locked operation.<sup>6</sup> Finally, section IV briefly discusses other aspects of high power microwave propagation in plasmas. Additional issues are discussed in two appendixes.

## II Propagation of a Chirped Pulse

In this section, we analyze the propagation of a pulse whose frequency varies with time in a dispersive medium. The application envisioned is high power microwave pulse compression in a plasma filled waveguide. Imagine that a pulse enters the guide at  $t=0$  and that the guide has length  $L$ . If the first part of the pulse has group velocity  $v_g(0)$ , the time to emerge out the other end of the guide is  $T=L/v_{g0}$ . For a portion of the pulse entering at a later time  $t$ , the condition that it also emerge at time  $T$  is

$$T = L/v_{g0} = t + L/v_g(t) \quad (1)$$

where  $v_g(t)$  is the group velocity of the portion of the pulse that entered at time  $t$ . For pulse compression  $v_g$  must be an increasing function of time to enable the later parts of the pulse to "catch up" to the earlier portions. Notice that while the frequency must be well controlled as function of time, there is no need for the amplitude to be particularly well controlled. Thus, utilization of this scheme is less demanding than other methods for short pulse propagation requiring both frequency and amplitude control. This means of pulse propagation is shown schematically in Fig. 1. In Appendix A, we show that a stationary phase analysis of the signal propagation verifies this analysis. There, it is shown that pulse compression by a factor of fifty to one hundred is theoretically possible.

Since the group velocity is a specified function of frequency, the variation of group velocity with time implies a variation of frequency with time, or in other words a "chirp" to the pulse. For high-powers, a plasma loaded waveguide is very attractive because, first, the plasma can withstand



very high power propagation, and second, the plasma can be very dispersive near one of its natural frequencies, implying short system length. In the following two subsections we consider the propagation of microwaves in an unmagnetized plasma near the plasma frequency, and in a strongly magnetized plasma near the cyclotron frequency. The dispersion relations for these two cases are shown in Fig. 2.

#### A. Unmagnetized Plasma

If a uniform, unmagnetized plasma fills the wave guide, the dispersion relation is

$$\omega^2 = k^2 c^2 + \omega_p^2 + \omega_{co}^2 \quad (2)$$

where  $\omega_p$  is the plasma frequency and  $\omega_{co}$  is the vacuum guide cutoff frequency. This is the same as an empty guide, except that the effective cutoff frequency is significantly increased with a plasma fill from  $\omega_{co}$  to  $(\omega_p^2 + \omega_{co}^2)^{1/2}$ . The transverse mode structure is also similar to that of an empty guide. The plasma filled guide, as a dispersive medium, offers the advantage over the vacuum guide in that if  $\omega_p \gg \omega_{co}$ , the medium is highly dispersive far above the vacuum fundamental wave guide mode cutoff frequency. Thus mode competition can be eliminated at high frequency and even with large wave guide radius (and therefore large power).

Let us now imagine that a frequency chirped wave enters a plasma filled waveguide. The plasma density is chosen such that the input wave frequency is only slightly above the plasma frequency. When the wave enters the plasma region, its wave number changes significantly. Thus, if no precautions are taken, a large part of the signal will be reflected. However, as shown in

Appendix B, the use of a transition cell of intermediate plasma density can eliminate this reflection. Using the fact that the group velocity is given by  $V_g = kc^2/\omega$ , and that  $\omega$  does not change as the signal enters the plasma, one can use Eqs. (1 and 2) to calculate the optimum frequency chirp for a 3 meter guide. The result is shown in Fig. (3) for a 20 nsec input radiation pulse. At the earliest time, the frequency is 3% above the plasma frequency. By moving the frequency further above the plasma frequency, the group velocity increases, so the optimum chirp is a frequency which increases with time, as expected.

#### B. Magnetized Plasma

A magnetized plasma is very dispersive near the cyclotron frequency. Moreover, the waves near  $\Omega$  have very short wavelengths and slow group velocities. Because the group velocity becomes so small and the dispersion so great, very short guides can be used for pulse compression. Since the wave number in the plasma times the guide radius can now be assumed large, we ignore neglect transverse effects and consider only parallel propagation.

For right hand circular polarization, the dispersion relation is<sup>7</sup>

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{c^2(\omega - \Omega)}, \quad (3)$$

where  $\Omega$  is the nonrelativistic electron cyclotron frequency. Notice that if  $\omega < \Omega$ ,  $k$  increases from  $\omega/c$  to some much larger value as the plasma density is raised. At no intermediate density does it go through zero or infinity. Thus, if the density is adiabatically increased from zero, the wave will propagate into the plasma with very little reflection. Alternatively, one could use a plasma cell of intermediate density to reduce reflection as

discussed in Appendix B. Once inside the plasma, (where  $\omega \lesssim \Omega$ ) the wave propagation is dominated by the plasma and

$$k^2 \approx \frac{\omega_p^2 \Omega}{c^2 (\Omega - \omega)}, \quad (4)$$

and

$$v_g \approx 2c \frac{(\Omega - \omega)^{3/2}}{\omega_p \Omega^{1/2}}. \quad (5)$$

Then using Eqs. (1 and 5), one can again determine the optimum frequency chirp. We assume X Band pulse so  $\omega \approx \Omega = 6 \times 10^{10}$ . Assuming  $\omega_{pe} = \Omega/2 = 3 \times 10^{10} \text{ cm}^{-3}$ ,  $\omega(t=0) = 0.95 \Omega$ , and a 1 meter long guide, the required pulse frequency as a function of time is shown in Fig. (4). Notice that the chirp needed is now a decreasing function of time.

### III. Pulse Compression of a Fixed Frequency Source

In the previous section we discussed high power pulse compression of a chirped pulse in a dispersive line. It is also possible to compress a fixed frequency pulse using a very similar technique. To make up for the fact that the pulse is fixed frequency, it is necessary to change the properties of the dispersive medium in time. This will induce a frequency chirp, which then allows for pulse compression. For instance, if while the pulse is entering the magnetic field changes in time, then each element of the pulse sees a different time history. A calculation of the optimum time history of, say, the magnetic field is much more subtle than a calculation of the frequency chirp (Eq. (1)). The Section is divided into three subsections, in which we first we present a general formulation, and then apply it to magnetized and unmagnetized plasmas.

#### A General Forumulation.

Here we present a formulation of the problem which allows for a straightforward numerical calculation of  $\phi(t)$  (or whatever the time varying parameter is). Then we give rough estimates for two cases; first, the strongly magnetized plasma, which is dominated by the time change of magnetic field; and second, the weakly magnetized plasma which is dominated by the change in density.

Let us imagine that the plasma begins at  $x=0$ . When the pulse propagates from just outside the plasma to just inside it, the frequency does not change because there has been no abrupt temporal change. The parallel wave number does change, however, because of the sharp spatial discontinuity. Once the pulse has entered the plasma it sees no additional spatial variation, since we assume a uniform plasma, so  $k$  is constant. But, since the medium does vary

slowly in time, the wave frequency,  $\omega$  also changes in time as the pulse propagates through the guide.

The problem now is to determine the optimum time variation of the medium for pulse compression. Let us say that the dispersive guide has length  $L$  and that the pulse first enters at time  $t=0$ . In vacuum, the pulse length is  $t_1$ , so that the medium, which varies only during the pulse input, varies between times  $t=0$  and  $t=t_1$ . Time  $T$  after time  $t_1$ , the medium has no additional time variation. We assume that at time  $t_1 + T$ , all parts of the pulse reach the end of the dispersive medium (i.e. they travel a distance  $L$ ). The group velocity of a portion of the pulse then is a function of two parameters, the time the pulse first entered,  $t_0$  (second index) and the actual time  $t$  (first index). Note that since we assume no variation of the medium for  $t > t_1$ ,  $V_g(t, t_0) = V_g(t_1, t_0)$  for  $t > t_1$ . Then the condition that all elements of the pulse reach the end of the dispersive line at the same time is

$$L - \int_{t_0}^{t_1} V_g(t, t_0) dt = T V_g(t_1, t_0) \quad (6)$$

where in Eq. (6),  $t_0$  takes on all values between 0 and  $t_1$ .

Now the time dependence of  $V_g(t, t_0)$  (as a function of two variables) is in fact characterized by the time variation of a single parameter, say  $\phi(t)$  (a function of one variable). The problem is to solve for  $\phi(t)$  such that Eq.(6) is satisfied. The key is to start at the last point  $t_0 = t_1$  and work backwards towards  $t_0=0$ . That is the microwave pulse which enters at  $t_0=t_1$ , sees no time variation, while the early part of the pulse which enters at  $t_0=0$ , sees the maximum amount of time variation. For  $t_0=t_1$ , Eq. (6) gives the result  $V_g(t_1, t_1) = L/T$ . This solves for  $\phi(t=t_1)$ . Let's divide the time interval from  $t_0=0$  to  $t_0=t_1$ , into  $N$  subintervals, with  $n=0$  at  $t_0=t_1$ , and  $n=N$

at  $t_0=0$ . The time interval between steps is  $\Delta = t_1/N$ .

We already know  $\phi$  at  $n=0$  (that is,  $\phi(t=t_1)$ ). To proceed, use induction. Imagine  $\phi$  is known up to  $n_j$  then calculate  $\phi(n+1)$ . To start, use Eq. (6) for the part of the pulse which enters at time  $t_0=t_1-(n+1)\Delta$ . This gives

$$L - \sum_{j=0}^{n+1} \Delta V_g(t_1 - j\Delta, t_1 - (n+1)\Delta) = T V_g(t_1, t_1 - (n+1)\Delta). \quad (7)$$

We assume that  $\phi(j)$  is known for  $0 \leq j \leq n$ . Thus Eq. (7) is a relation to solve for the single quantity  $\phi(n+1)$ .

To solve for  $\phi(n+1)$  assume  $\phi(t)$  changes slowly between time steps so that  $\phi(n+1) = \phi(n) + \delta\phi$ . For  $0 \leq t \leq t_1$ , time is a single-valued function of  $\phi(t)$ . Thus,  $V_g(t, t_0)$  can be expressed as  $V_g(x, y)$  where  $x = \phi(t)$  and  $y = \phi(t_0)$ . Then assuming the variation from one time step to the next is small,

$$V_g(t_1, t_1 - (n+1)\Delta) = V_g(t_1, t_1 - n\Delta) - \left. \frac{\partial V_g}{\partial y} \right|_n \delta\phi \quad (8)$$

correct to order  $\delta\phi$ , where the notation  $\left|_n$  means that  $x$  is evaluated at  $j=0$  and  $y$  is evaluated at  $j=n$ . Equation (8) is used in the right hand side of Eq. (7) to replace  $V_g(t_1, t_1 - (n+1)\Delta)$ . Now we expand the left hand side of Eq. (7) to order  $\Delta$  to express the change from the previous time step, where both sides were equal. There are two contributions to this change; first, there is an additional time step in the summation; and second, each element in the summation is altered because the pulse entered slightly earlier. Combining

both effects, Eq. (7) becomes

$$\begin{aligned}
 & -\Delta V_g(t_1 - (n+1)\Delta, t_1 - (n+1)\Delta) + \sum_{j=0}^n \Delta \left. \frac{\partial V_g(x, y)}{\partial y} \right|_j \delta \Omega \\
 & = -T \left. \frac{\partial V_g}{\partial y} \right|_0 \delta \Omega
 \end{aligned} \tag{9}$$

Finally, using the fact that  $V_g(t_1 - (n+1)\Delta, t_1 - (n+1)\Delta)$  is multiplied by  $\Delta$  in Eq. (9), so that correct to order  $\Delta$  it can be replaced by

$V_g(t_1 - n\Delta, t_1 - n\Delta)$ , Eq. (9) gives us the (desired) recursion relation for the change in cyclotron frequency in time, i.e.,

$$\frac{\delta \Omega}{\Delta} = \frac{d\Omega}{dt} = \frac{-V_g(t_1 - n\Delta, t_1 - n\Delta)}{-T \left. \frac{\partial V_g}{\partial y} \right|_0 + \sum_{j=0}^n \Delta \left. \frac{\partial V_g}{\partial y} \right|_j} \tag{10}$$

Note that the derivative of  $\Omega$  with respect to time at  $t$  depends on the entire previous history of  $\Omega$  from  $t_1$  to  $t$ . Since we know  $\Omega(t_1)$ , and  $V_g$  is a known function of  $x$  and  $y$ , Eq. (10) can be used to calculate  $\Omega$  backwards from  $t=t_1$  to  $t=0$ . At each iteration, the entire history over the known region  $\Omega(t_1 > t > t_1 - n\Delta)$  is used to calculate the rate of change of  $\Omega$  with respect to  $t$  at  $\Omega(t_1 - n\Delta)$ . This allows us to calculate  $\Omega$  at the next step,  $n+1$ , and so on until we are at  $t_0=0$ . The same procedure holds for other time varying plasma parameters in lieu of  $\Omega$ , the cyclotron frequency; for example the plasma frequency  $\omega_p$  follows a relationship similar to Eq. (10) if the plasma density is varied instead of the magnetic field. We will show an example of this later.

In the remainder of this section, we give approximate calculations relevant to pulse compression from density changes in unmagnetized plasmas, and from magnetic field changes in magnetized plasmas.

## B Unmagnetized Plasma

A fixed frequency wave with frequency  $\omega$  enters the plasma region at  $x=0$ . The wave number for the part of the pulse that enters at time  $t_0$ , is given by

$$k^2(t_0) = \frac{\omega^2 - \omega_p^2(t_0)}{c^2}. \quad (11)$$

The wave number remains constant at all subsequent times because once the pulse enters the plasma it is in a homogeneous medium. However, the frequency does change with time as the signal propagates because the density changes in time. The frequency is then a function of two variables,  $t$ , the actual time, and  $t_0$ , the time that portion of the pulse initially entered the plasma. Using the fact that  $k^2$  is constant in Eq. (11) we find a relation for  $\omega^2(t, t_0)$ ,

$$\omega^2(t, t_0) = \omega^2 + \omega_p^2(t) - \omega_p^2(t_0). \quad (12)$$

Since the group velocity is  $V_g$  is given by the local dispersion relation, Eq. (2), in the varying medium,  $V_g = kc^2/\omega$ , we find that the group velocity is also a function of the two variables,  $t$  and  $t_0$  i.e.,

$$V_g(t, t_0) = c(\omega^2 - \omega_p^2(t_0))^{1/2} / (\omega^2 + \omega_p^2(t) - \omega_p^2(t_0))^{1/2}. \quad (13)$$

Letting  $\omega_p^2(t) = x$  and  $\omega_p^2(t_0) = y$ , then Eq. (13) becomes

$$V_g(x, y) = c(\omega^2 - y)^{1/2} / (\omega^2 + x - y)^{1/2} \quad (14)$$



Now, the derivation leading up to Eq. (10) applies except that instead of  $\delta\Omega$  and  $d\Omega/dt$ , use  $\delta\omega_p^2$  and  $d\omega_p^2/dt$ . Thus Eqs.(14) and (10) give the optimum density time variation for pulse compression..

To continue we give a crude estimate of the required density change. If the pulse length of the entering pulse is  $t_1$ , the group velocity of the portion of the pulse which entered the plasma last is

$$v_g(t_1, t_1) = c(\omega^2 - \omega_p^2(t_1))^{1/2}/\omega. \quad (15)$$

At time  $t_1$  the group velocity of the part of the pulse that entered at  $t=0$  is

$$v_g(t_1, 0) = c(\omega^2 - \omega_p^2(0))^{1/2}/(\omega^2 + \omega_p^2(t_1) - \omega_p^2(0))^{1/2}. \quad (16)$$

$v_g(t_1, t_1)$  must be greater than  $v_g(t_1, 0)$  in order for there to be pulse compression. For this to be true,  $\omega_p^2(t_1) < \omega_p^2(0)$ ; the density must be a decreasing function of time.

Figure (5) shows a calculation of the optimum density as a function of time. The vertical axis is  $\omega_p^2$  in  $\text{sec}^{-2}$ . We assumed  $t_1=20$  nsec,  $T=60$  nsec and  $\omega = 6 \times 10^{10}$ . The density is a decreasing function of time as discussed.

Since the required density drop is over a very short time scale, it is not feasible to rely on recombination or attachment to reduce the density. The more viable approach to reduce  $\omega_p^2$  is to use an inverse theta pinch. Imagine that at  $t=0$  the plasma is weakly magnetized so that  $\omega_p \gg \omega_c$ . Thus the magnetic field could confine the plasma, but it would have very little effect on wave propagation if  $\omega \gg \omega_c$ . For instance in our X-band pulse compression example, a magnetic field of only 500 G ( $\omega_c = 9 \times 10^9$ ) could confine a plasma having  $\omega_p \approx 6 \times 10^{10}$ . If the magnetic field changes quickly, the density

should drop due to the plasma being frozen in the field. The variation in density should be approximately what is required if the magnetic field were reduced by 20% over the 20 ns pulse time. The flux change in the plasma over 20 nsec implies a voltage of only one kilovolt (for a single turn coil), an energy change of 0.1 Joule per centimeter and a power investment of 5 Megawatts per centimeter. These values are well within the range of modest pulsed power technology.

### C. Magnetized Plasma

We now consider the second approach to pulse compression in a time-varying dispersive plasma, the propagation of right hand circularly polarized wave with frequency near the electron cyclotron frequency. In this case we assume that the cyclotron frequency changes with time. However, as we have just seen, a change in cyclotron frequency implies a corresponding change in density, since over the nanosecond time we consider, the plasma is frozen into the field (or  $\omega_p^2 / \Omega$  remains constant).

As stated in Section IIIB, when the wave enters the plasma at time  $t_0$  at fixed frequency  $\omega$  its wave number becomes

$$k^2(t_0) = \frac{\omega_p^2(t_0) \omega}{c^2(\Omega(t_0) - \omega)}, \quad (16)$$

assuming  $kc/\omega \gg 1$ . As in the previous subsection, we then determine

$$\omega(t, t_0) = \frac{\frac{\Omega(t) \omega_p^2(t_0) \omega}{\Omega(t_0) - \omega}}{\omega_p^2(t) + \frac{\omega_p^2(t_0) \omega}{\Omega(t_0) - \omega}}. \quad (17)$$

Using the fact that the flux is frozen into the plasma and  $\omega_p^2/\Omega$  is constant, Eq. (17) becomes

$$\omega(t, t_0) = \frac{\omega}{1 + \omega/\Omega(t) [1 - \Omega(t)/\Omega(t_0)]} \quad (18)$$

A similar calculation for the group velocity gives the result

$$v_g(t, t_0) = \frac{2c}{\omega_{po} \Omega(t)} \left( \frac{\omega}{\Omega(t_1) - \omega} \right)^{1/2} \left( \Omega(t) - \frac{\omega}{1 + \frac{\omega}{\Omega(t)} [1 - \frac{\Omega(t)}{\Omega(t_0)}]} \right)^2, \quad (19)$$

where  $\omega_{po}$  is the plasma frequency at  $t=0$ . With  $x = \Omega(t)$  and  $y = \Omega(t_0)$ , Eq. (19) above is in the form given in Eq. (10), so that Eq. (10) can be used to calculate the optimum time dependence of  $\Omega(t)$ . One can now show with straightforward manipulations that if  $V(t_1, t_1) > V(t_1, 0)$ , then  $\Omega(t_1) > \Omega(0)$ . Thus for pulse compression, the magnetic field must be compressed. From Eqs. (10) and (19), one can calculate the optimum dependence of the magnetic field in time. Figure 6 shows the calculated dependence of  $\Omega(t)$  for a 50 nsec pulse of microwave radiation with  $\omega = 6 \times 10^{10}$  for a guide with  $T = 150$  nsec.

#### IV. Other Plasma Effects

In this section we briefly discuss some additional possibly deleterious, effects of the plasma on pulse propagation and compression. We start with a discussion of plasma wave damping, then discuss bleaching, and finally speculate on the effects of parametric instabilities.

##### A Wave Damping in the Plasma

We begin with a study of microwave damping (absorption) in the unmagnetized plasma. Since the phase velocity is very large and there is no magnetic field (or very weak field), then only collisional damping will be important (not Landau or cyclotron damping). The dispersion relation for a transverse wave in an unmagnetized plasma becomes<sup>7</sup>

$$\frac{k^2 c^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}, \quad (20)$$

where  $\nu$  is the electron momentum transfer collision frequency. If the oscillating velocity of electrons in the wave field,  $V_{os}$ , is small compared to the electron thermal velocity, then  $\nu$  is given by

$$\nu^{-1} = 3.4 \times 10^5 T_e^{3/2} / n_e \lambda, \quad (21)$$

where  $\nu$  is in  $\text{sec}^{-1}$ ,  $T_e$  is in electron volts,  $n_e$  is the electron density in  $\text{cm}^{-3}$  and  $\lambda$  is the Coulomb logarithm (usually between 5 and 10). If the microwave field amplitude is large, and the oscillating velocity becomes comparable to or greater than the electron thermal velocity, then  $T_e$  is enhanced by the oscillating motion of the electrons and  $\nu$  is further reduced. We will, as a guide, replace  $T_e$  with  $T_e + mv_{os}^2$  in the event the

latter term is significant.

For example, if  $\omega$  is 3% above the plasma frequency and  $\omega/c = 2 \text{ cm}^{-1}$ , as with our X-band example, then the wave absorption e-folding length is 5 meters in a  $T_e = 1 \text{ eV}$  plasma. The actual damping length would be considerably longer because the temperature probably would be greater than 1 eV at the 100MW to 1GW microwave power level in a 10 cm radius waveguide; also  $V_{os}$  may be comparable to the thermal velocity. Thus, damping of the wave by the plasma should not be a significant loss mechanism for the unmagnetized case.

We now consider the magnetized case. As we will see, conditions are more stringent, but the magnetized plasma pulse compression still remains a viable scheme. In this case the wave now has low phase velocity and Landau and cyclotron damping must also be considered. We first consider collisional damping. The dispersion relation becomes<sup>7</sup>

$$k^2 = \frac{\omega^2}{c^2} - \frac{\omega_p^2}{(\omega - \Omega - i\nu)} \quad (22)$$

For the example of Section II B, with  $\omega \approx \Omega = 6 \times 10^{10}$  and  $\omega_{pe} = 3 \times 10^{10}$ , and  $\nu = 7.5 \times 10^6$  for a 1 eV plasma. Also if we take  $\omega - \Omega \approx 0.08\Omega$  which is a reasonable average value, then  $k_r \approx 4/3 \text{ cm}^{-1}$  and  $k_i \approx 5 \times 10^{-3} \text{ cm}^{-1}$ . Thus, the damping length is about 2 meters, longer than the dispersive medium. Hence collisional damping is negligible, particularly if the temperature is greater than 1 eV.

Next, we consider cyclotron damping. This could be important if  $(\Omega - \omega)/k \sim V_e$ , where  $V_e$  is the electron thermal velocity. For the example we have been considering, cyclotron damping is negligible as long as the temperature is much less than 300 eV. Thus neither cyclotron damping nor collisional damping should be important for the magnetized plasma pulse

compression. Landau damping will not be much less important than cyclotron damping because  $\omega/k \gg \omega)/k$ .

### B. Bleaching Effects

The damping and absorption of the wave in the plasma can in some circumstances be beneficial. Imagine that the wave, either before or after compression, is preceded by an undesirable prepulse. As the wave is sent through a low temperature (and perhaps higher density) plasma cell, the prepulse collisionally damps out, but in doing so, the absorbed prepulse energy would heat the plasma to a sufficiently high temperature that the main pulse which follows would be undamped. The calculation of the plasma parameters necessary for bleaching as a function of the prepulse is straight forward and will not be presented here.

### C. Parametric Instabilities

Whenever a high power beam of electromagnetic radiation passes through a plasma, there is a potential for parametric instability. These are important when the oscillating velocity of electrons in the wave electric field is comparable to, or greater than, the electron thermal velocity. This oscillating velocity in an unmagnetized plasma is given by

$$v_{os} = 2.5 \times 10^5 I^{1/2} \lambda \text{ cm/sec} , \quad (23)$$

where the wave irradiance  $I$  is given in  $\text{W/cm}^2$  and wavelength  $\lambda$  is in cm. The ratio of oscillating energy to thermal energy is

$$W_{os}/W_{th} = \frac{1.8 \times 10^{-5} \lambda^2 I}{T_e} , \quad (24)$$

where  $T_e$  is in ev.

The scope of parametric instabilities is very extensive and we can give here only a very superficial discussion.<sup>8</sup> We consider first the unmagnetized case. The possible parametric process is the decay of the microwave (pump) into an electron plasma wave oscillation and an ion wave (density oscillation). Since the growth rate usually scales as  $W_{os}/W_{th}$  (which can be larger than unity), these parametric instabilities are a potential problem. However, in the pulse compression case, it is power, not power density which is important. Thus the power density can always be reduced by taking a larger guide radius until one is below threshold for parametric instability.

Moreover, parametric instabilities depend on a precise phase relation between the pump wave and the decay waves. In this respect, the frequency chirp of the pump wave, can be a strong stabilizing effect. An analogous problem has been worked out in spatially varying media. For the instability to occur, the pump wave frequency and wave number  $\omega, k$  must be related to the decay wave frequencies  $\omega_1, \omega_2$ , and wave numbers  $\underline{k}_1, \underline{k}_2$  as

$$\omega = \omega_1 + \omega_2, \text{ and} \quad (25a)$$

$$\underline{k} = \underline{k}_1 + \underline{k}_2. \quad (25b)$$

In spatially varying but steady state media, Eq. (25a) is satisfied easily, but Eq. (25b) is only satisfied exactly locally at, say,  $x=0$ . Growth can then occur in a small region about  $x=0$  where the wave number mismatch is not too great<sup>8</sup>. If we say that in the spatially varying case

$$k - k_1 - k_2 = Kx. \quad (26)$$

and if the growth rate of the parametric instability is  $\gamma$  and the group velocities of the decay waves are  $V_1$  and  $V_2$ , then the number of power e foldings is  $2\pi\gamma^2/K V_1 V_2$ . If about 10 e-folds are necessary for a significant effect, then the condition for the importance of parametric instabilities is that

$$\frac{2\pi\gamma^2}{K V_1 V_2} \gtrsim 10. \quad (27)$$

Now consider the extension of this analysis to the case of the chirped microwave pulse. We will work in the reference frame of the group velocity of this pulse, where the system is nearly time independent. If the frequency width is  $\Delta\omega$ , then the equivalent width in  $k$  is  $\Delta k \sim \Delta\omega/V_g$ , where  $V_g$  is the group velocity of the pump wave. If the length of the pulse is  $V_g t_p$ , where  $t_p$  is the pulse time, then the mismatch condition is given roughly by  $K \sim \Delta\omega/V_g^2 t_p$ . Since both decay waves are electrostatic, their group velocities are much less than that of the pump in the lab frame; therefore in pulse frame they are almost equal, i.e.,  $V_1 \approx V_2 \approx V_g$ . Thus, the condition for significant parametric growth is

$$\frac{\gamma^2 t_p}{\Delta\omega} > 1 \quad (28)$$

For our X-Band example,  $\Delta\omega \sim 6 \times 10^9$  and  $t_p \sim 2 \times 10^{-8}$  so decay into an ion wave will be important only if  $\gamma \gtrsim 6 \times 10^8$ , a value comparable to the ion plasma frequency  $\omega_{pi}$ . Since parametric instability growth rates rarely exceed  $\omega_{pi}$ , it does not seem likely that decay into an ion wave and an electron plasma wave will be a dominant process.



We now consider briefly other parametric instabilities specific to magnetized plasmas. One potential process which could be particularly dangerous is the decay of the right hand circularly polarized pump (microwave) pulse into a lower frequency sideband and a low frequency whistler wave. All three waves have nearly the same group velocity. The dispersion relation for the right hand circularly polarized wave is

$$\omega = \frac{k_c^2 \Omega}{\omega_p^2 + k_c^2}, \quad (29)$$

as long as  $k \gg \omega/c$ . If the pump wave has  $\omega \approx \Omega$  and  $k \gg \omega_p/c$ , then it can decay into a low frequency wave with

$$k_1 = \frac{2\omega_p^4}{c^4 k^3}, \text{ and } \omega = \frac{k_1^2 \Omega}{\omega_p^2}, \quad (30)$$

and a lower sideband with  $\omega_2 = \omega - \omega_1$ ,  $k_2 = k - k_1$ . The main effect of the beating of the two high frequency waves is to produce a force in the direction  $\underline{k}_1$ , which is along the magnetic field.<sup>9</sup> However, the low frequency whistler wave only has currents only perpendicular to B so there is no coupling. But, the sidebands should have small components of their wave number perpendicular to B, there could be a coupling proportional to  $k_1$ . This is an area for future study.

The final process we consider is the decay of the pump wave into two electron electrostatic waves. These have the dispersion relation

$$1 - \frac{\omega_{pe}^2}{\omega_1^2} - \frac{k_{1\parallel}^2}{k_1^2} - \frac{\omega_{pe}^2}{\omega_1^2 - \Omega^2} - \frac{k_{1\perp}^2}{k_1^2} = 0, \quad (30)$$

where the index one denotes the electrostatic wave. These modes must have

frequency less than that of the pump wave for the pump wave is to decay into them. Let us consider the regime  $\omega \sim \omega_{pe}$  (we have been considering  $\omega_{pe} \approx \omega/2$ ). For  $\omega_1 < \omega$ , the electrostatic waves only propagate if

$$0 < \omega_1 < \text{Min}(\omega_{pe}, \omega) \quad (31a)$$

or

$$0 < \omega_1 < \omega_{pe} \cdot \quad (31b)$$

Thus, as long as  $\omega_{pe} < \omega/2$ , the parameter range we have been considering, there can be no decay into two electron electrostatic waves.

A comprehensive study of the potential problems of parametric instability in our pulse compression schemes is very difficult; they will almost certainly restrict, in some way, the range of parameter space in which the plasma pulse compressor can operate. But, on first examination, it appears that the plasma dispersion pulse compression schemes look attractive, at least for the parameters we have chosen. A more definitive assessment of these high-power pulse compression concepts, and whether they can be made into practical devices will require more work, both theoretical and experimental.

#### Acknowledgment

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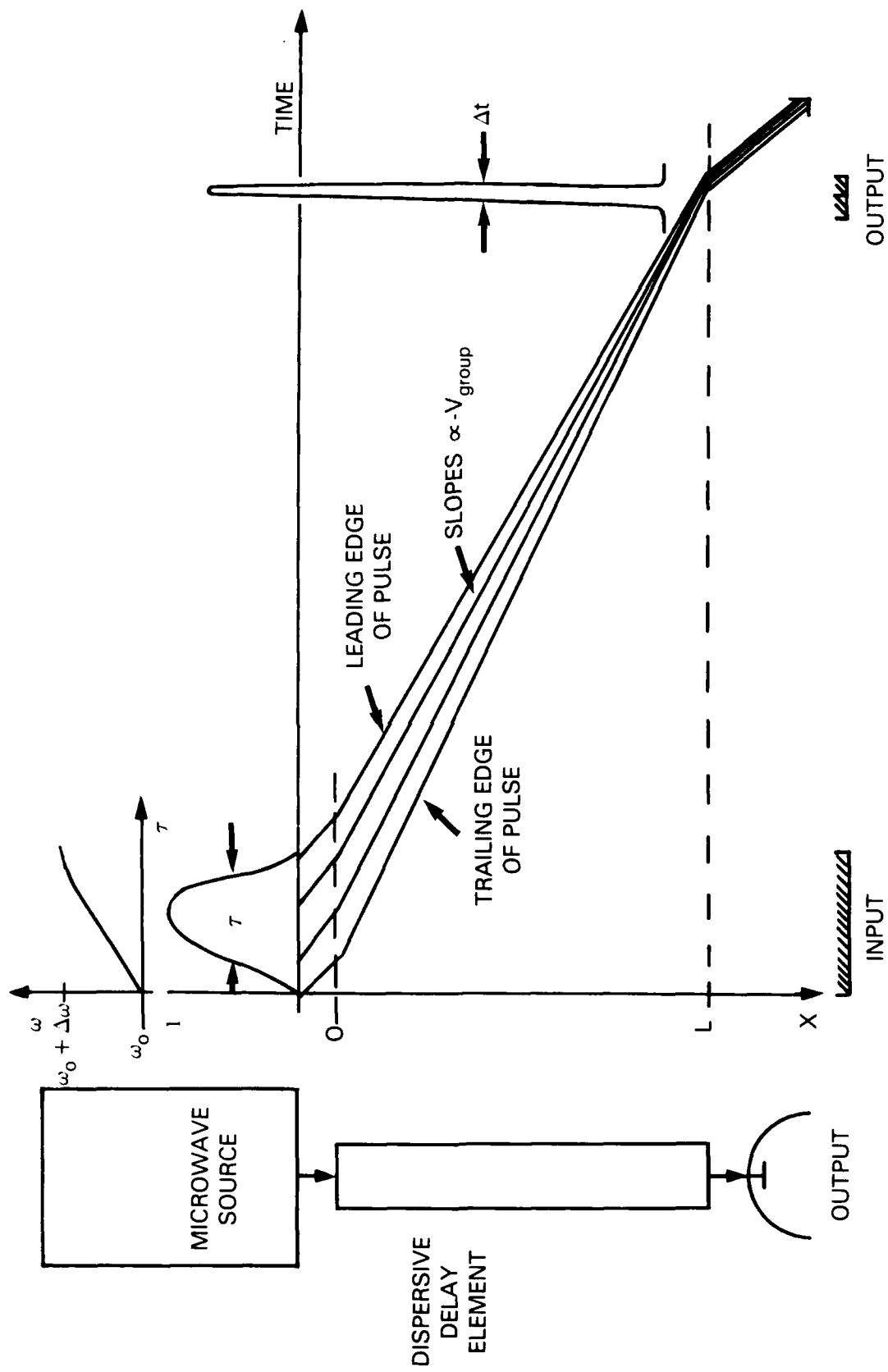


Figure 1. A schematic of the dispersion line pulse compression scheme.

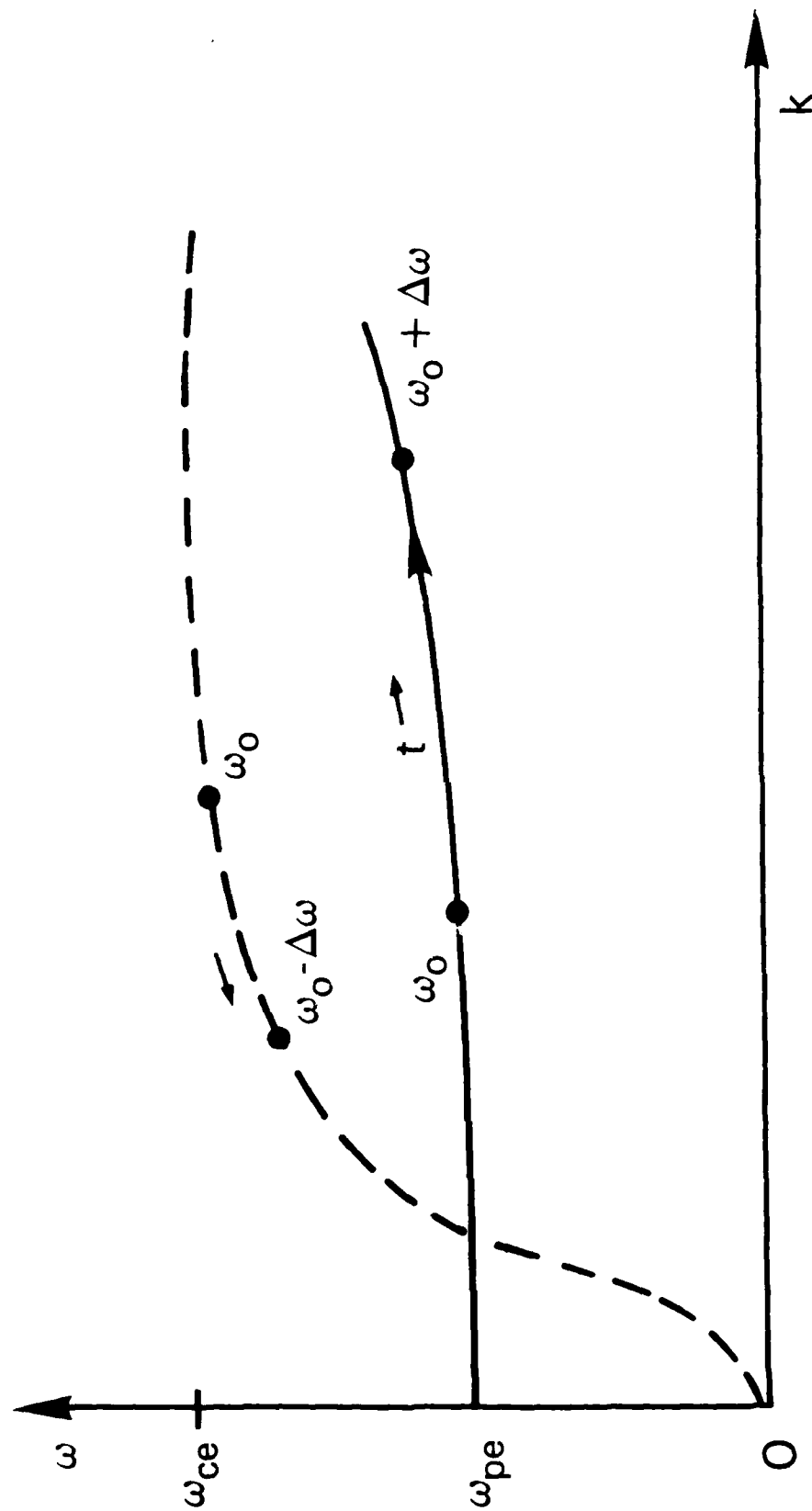


Figure 2. The  $(\omega, k)$  diagram for plasma loaded waveguide modes (----) and for right hand circularly polarized mode (-----).

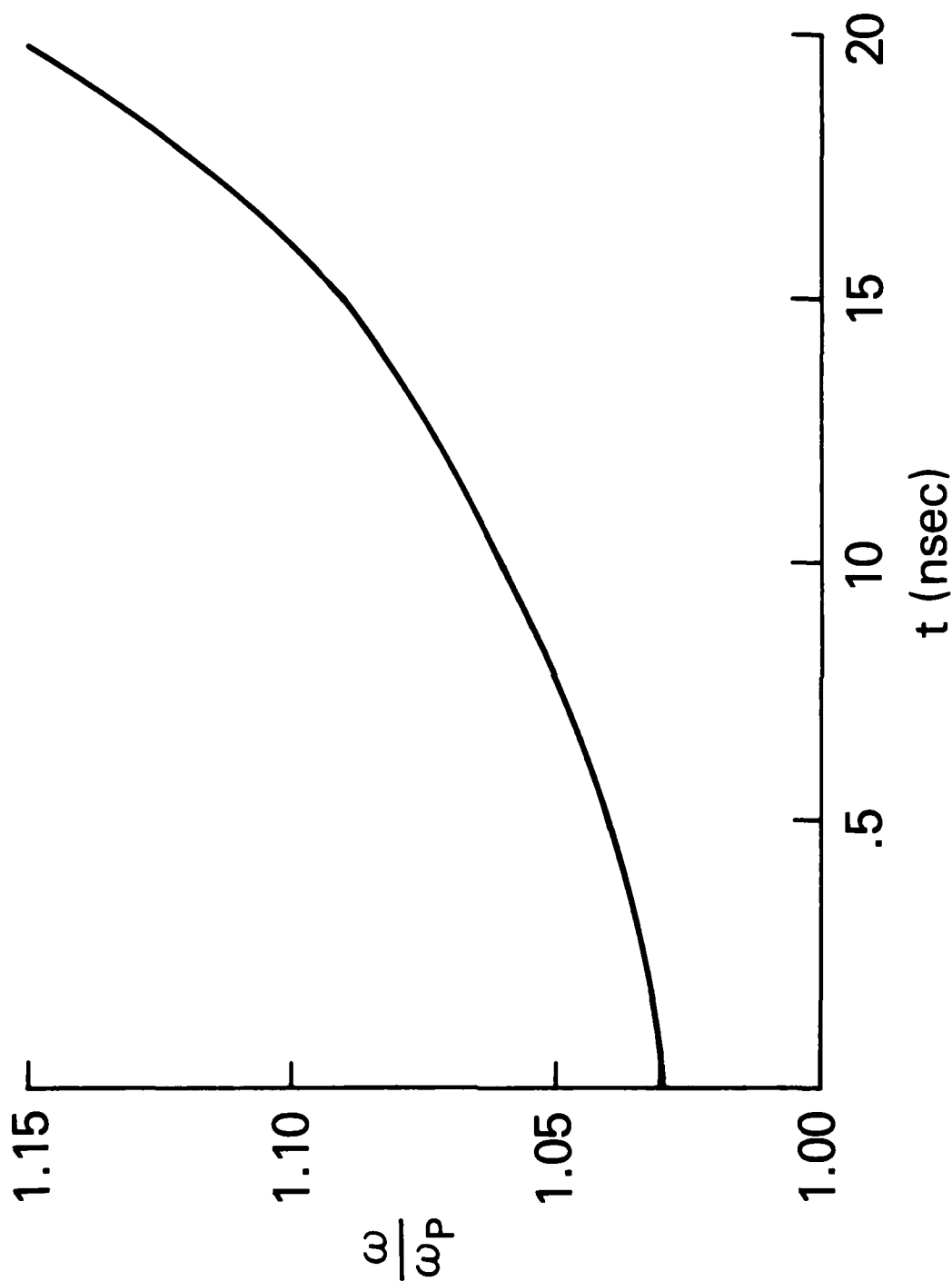


Figure 3. The frequency as a function of time for optimum pulse compression in a 3 meter guide using an unmagnetized plasma.

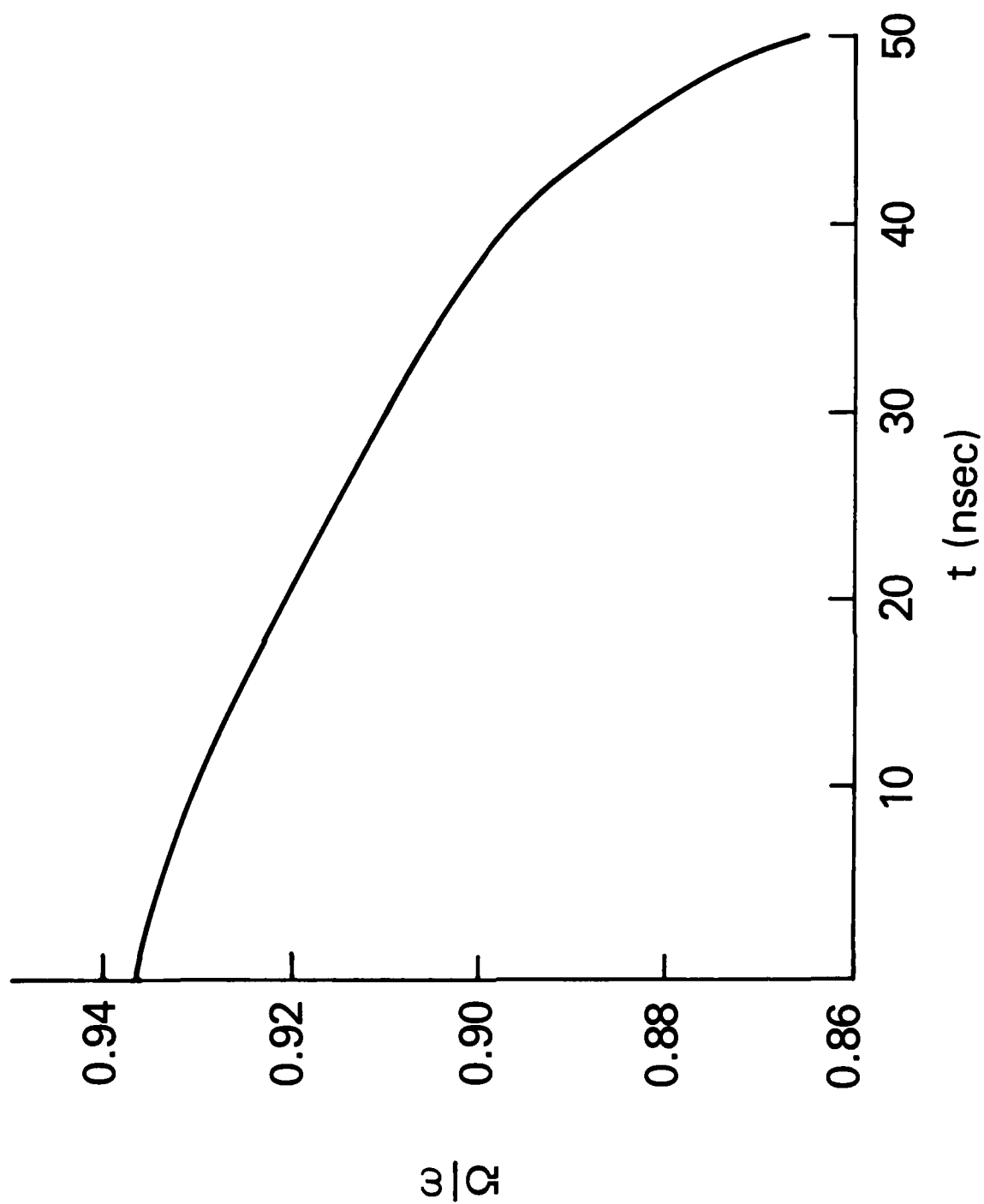


Figure 4. The frequency as a function of time for optimum pulse compression using a 1 meter magnetized guide.

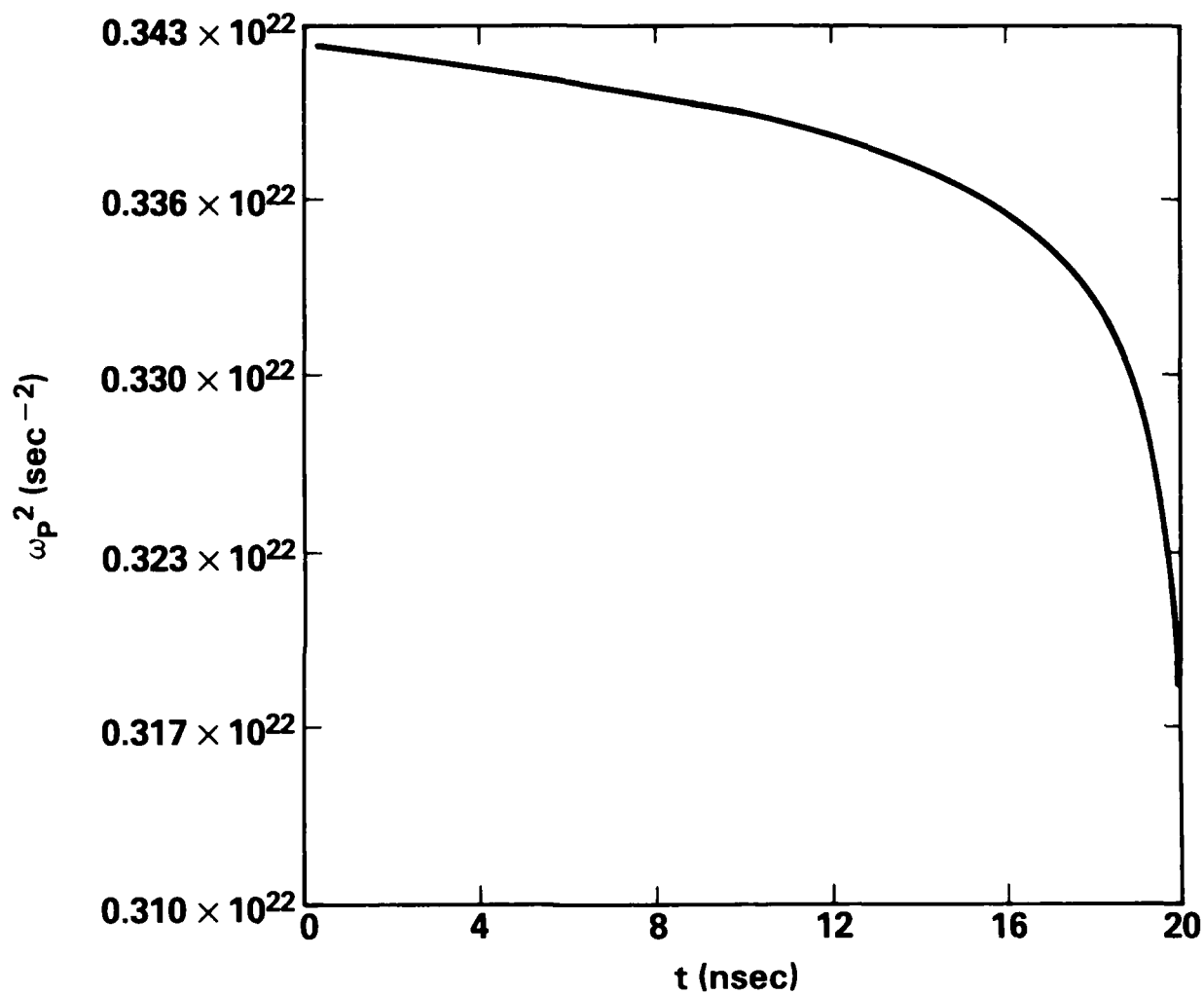


Figure 5. The optimum density as a function of time for

$\omega = 6 \times 10^{10} \text{ sec}^{-1}$ ,  $t_1 = 20 \text{ nsec}$ ,  $T = 60 \text{ nsec}$ . The vertical axis is plasma frequency squared.

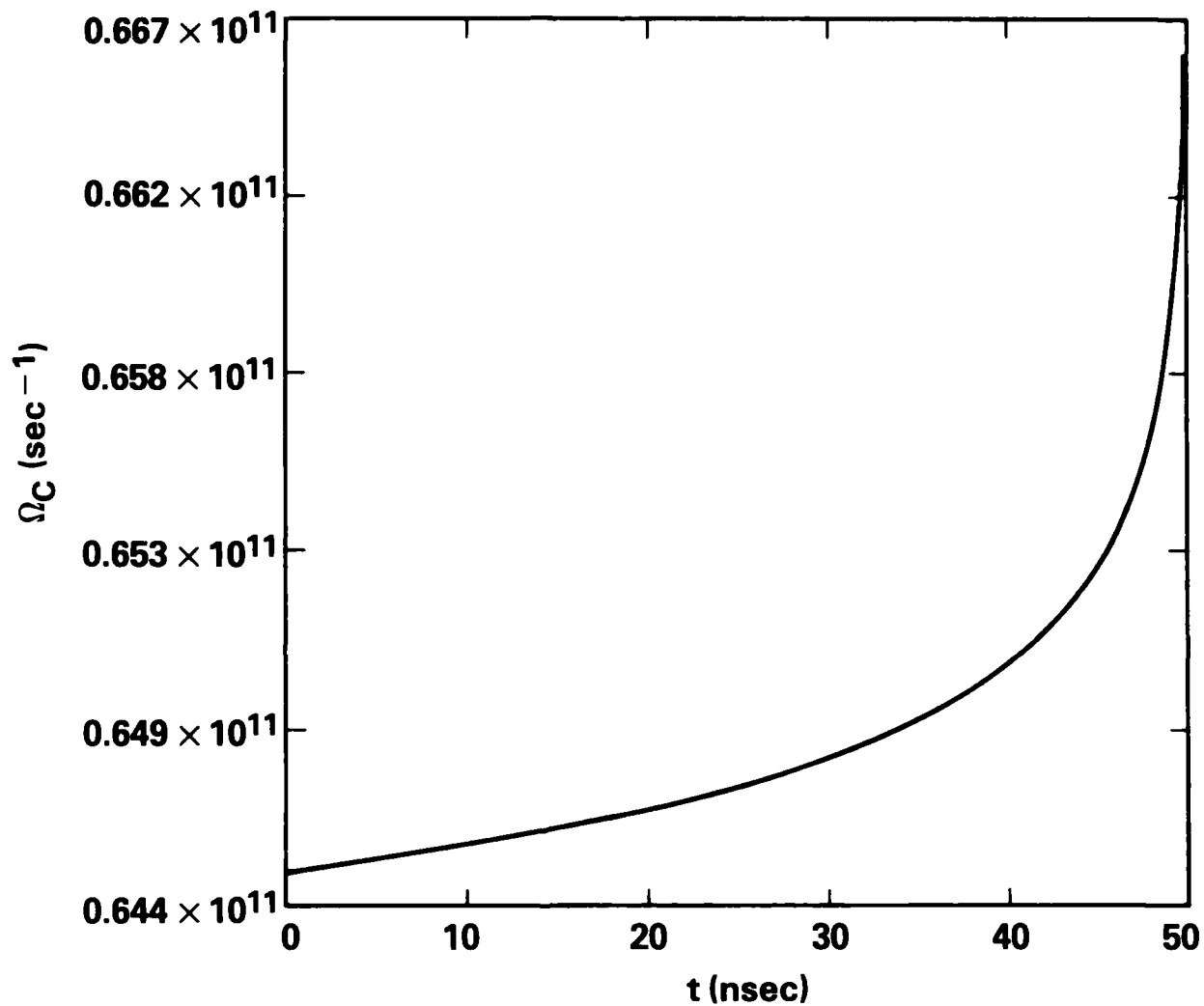


Figure 6. The optimum magnetic field as a function of time for  $\omega=6 \times 10^{10} \text{ sec}^{-1}$ ,  $t_1=50 \text{ nsec}$  and  $T=150 \text{ nsec}$ . The vertical axis is cyclotron frequency.



## Appendix A

### A Calculation of Pulse Compression

To calculate pulse compression more accurately, say the input wave is given by

$$E(t) = H(t) \exp -i \int_0^t \omega(t') dt' \quad (A1)$$

where H is an envelope function. To be specific, we will take

$$H(t) = E_0 \exp -\alpha^2 t^2, \quad (A2)$$

which, as we will see, makes the calculation somewhat simpler. The Fourier transform of the input wave is

$$E(\Omega) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} E(t) \exp i\Omega t \quad (A3)$$

To each Fourier component in time  $\Omega$  corresponds a spatial wave number  $k(\Omega)$ . Thus taking the inverse Fourier transform, we find that E, as a function of time and space is given by

$$E(t, z) = \frac{1}{2\pi i} \int d\Omega \int dt' E(t') \exp i\Omega t' - i\Omega t + iK(\Omega)z. \quad (A4)$$

To continue, we assume that the frequency variation is small so that

$$K(\Omega) = K_0 + K'(\Omega - \Omega_0) + \frac{1}{2} K''(\Omega - \Omega_0)^2. \quad (A5)$$

where a prime on a K variable signifies a derivative with respect to  $\Omega$ .

The  $\Omega$  integral can then be done analytically, yielding

$$E(t, z) = \frac{1}{2\pi i} \int dt' E(t') \exp i (\Omega_0(t' - t) + K_0 z) \quad (A6)$$

$$\times \left( \frac{2\pi}{K'' z} \right)^{1/2} \exp - \frac{i K'' z}{2} \left( \frac{K' z + t' - t}{K'' z} \right)^2.$$

To continue, we specify  $E(t')$  by Eqs. (A1) and (A2). This integral can be evaluation by the method of stationary phase. The exponent is given by  $i \theta(t', t, z)$  and the stationary phase point is given by

$$\frac{\partial \theta(t', t, z)}{\partial t'} = 2i\alpha^2 t' + \Omega_0 - \omega(t') - i \frac{K' z + t' - t}{K'' z} = 0 \quad (A7)$$

To see just what the stationary phase point means, consider first the case of  $\alpha=0$ . At  $\Omega = \omega(t')$ , the group velocity is  $(dK/d\Omega)^{-1}$ , or

$$V_g = [K' + K''(\omega - \Omega)]^{-1} \quad (A9)$$

Using this, we can show that  $\partial \theta / \partial t' = 0$  when

$$z - V_g(t')(t - t') = 0 \quad (A9)$$

Denoting the solution of Eq. (A7) by  $t' = t_s(z, t)$  the saddle point integral gives the result

$$E(t, z) = \frac{E_0 \exp i \theta(t_s, t, z)}{(K'' z \theta''(t_s, t, z))^{1/2}} \quad (A10)$$

where  $\theta''$  is  $\partial^2 \theta / \partial t'^2$ . The quantity in the denominator is given by  $[2i\alpha K'' z + K'' z \omega'(t) - 1]^{1/2}$ . Again, if  $\alpha = 0$ , one can show that the

denominator is proportional to  $\frac{\partial}{\partial t} (z/v_g)$ ; that is the derivative with respect to time of arrival time of the pulse. If the denominator is zero, all parts of the pulse arrive at  $z$  at the same time and there is maximum pulse compression. The limit of pulse compression is then determined by  $\alpha$ . The maximum increase in power, (that is  $E^2$ ) is  $(2\alpha^2 K'' z)^{-1}$ . Using the fact that at maximum compression,  $K'' z = \omega'(t)$ , we find that

$$\frac{P \text{ (compressed)}}{P \text{ (initial)}} \approx \frac{1}{2} \omega' / \alpha^2 \approx \frac{1}{2} \omega' t_p^2 \quad (\text{A11})$$

where  $t_p$  is the pulse time. If we consider our standard example of

$\omega = 6 \times 10^{10}$  and a 10% variation in  $\omega$  over a pulse time of 20 nsec, we find  $P_c/P_{in} = 60$ .

## APPENDIX B

Since a microwave pulse entering a plasma region will ordinarily suffer large reflection, it may be necessary to have a plasma cell of intermediate density at the input and output. Let region I be vacuum, and having frequency and wave number  $(\omega, k_1)$ . Let region III be the plasma having  $(\omega, k_3)$ . In between (region II) assume a transition plasma cell of intermediate density, length  $L$  and having  $(\omega, k_2)$ . Assume that the boundary condition is that both the electric field and its first derivative are continuous at each transition point.

Assuming no reflected wave, the input electric field is  $E_1 \exp i k_1 x$ . In the intermediate region it is  $E_{2+} \exp i k_2 x + E_{2-} \exp -i k_2 x$ . In the third region,  $E_3 \exp i k_3 x$ . Matching the boundary conditions at each interface,  $x=0$  and  $x=L$  gives four homogeneous equations for four unknown  $E_1, E_{2+}, E_{2-}, E_3$ . The condition for non trivial solution is that the determinant of the coefficient vanishes. Setting up the determinant and evaluating it, it is a straightforward matter to show that the density of the intermediate plasma is determined so that

$$k_2^2 = k_1 k_3 \quad (B1)$$

and the length of the region is determined so that

$$\exp i k_2 L = -1. \quad (B2)$$

Thus by choice of an appropriate transition cell, reflection can be eliminated.

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